Oslo Special Year kick-off conference 2018/8/27-29

my talk 3^{30} on 8/27

title "Some projects I'd like to work on"

Intro. Actually, it's one project with several parts — < UniMath development >

That sort of works, because Vladimir, in one of his 5 talks on it, gave 3 definitions of UniMath: (2017 July)

(1) a univalent foundation of mathematics

(2) a formal language for implementing it

(3) a github repository containing a library of proofs, based on Coq, ~~which aims~~

It aims to:

(0) formalize ~~known~~ a substantial ~~part~~ body of mathematics

as VV pointed out, we need a better substrate to support

(1) resizing

~~(2) $(\infty,1)$-categories~~

(2) computation of univalence (cubical TT)

& (3) $(\infty,1)$-categories (simplicial TT Why not?)

simplicial semantics for HIT's.

(0) (4) — my paper on KnF by gen's & relations

def. ex. category, mention Quillen's def'n, (K_0 first)

describe binary chain complexes, l.e.s.

— also transfinite recursion for types

$(X$ w.o. set, $P : \prod_{x:X}$ Type,

$r : \prod_{x:X} \left(\prod_{y:X} (y < x \to P\ y) \right) \to \prod_{x:X} P\ x)$

— also dependent recursion for $S^1 := B\mathbb{Z}$

$(X : S^1 \to$ Type, $x : X(\text{base})$,

$\ell : \text{loop} \# x = x \to \prod_{t:S^1} X(t))$

(1) Resizing

UniMath current disables universe checks.

Coq now has "universe polymorphism".

So we can implement propᵢl resizing this way

Definition ~~resize@{} (T : Type@{j}~~

resize@{i j | i<j} (T : Type@{j}) :

isaprop T → Type@{i} := λ_, T.

Mention we need Coq not to convert some universe inequalities to equalities. Try fixing Coq or waiting for Matthieu

~~(2) If cubical TT works for computation, so should simplicial TT, so I build on exploration of~~

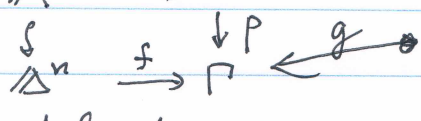
(4) Lumsdaine-Shulman point out the need to be able to make the pre-suspension of a fibrant family into a fibration in a way compatible w. base change

Here is a conjecture to focus on.

def. quasi-fibration in topology [Dold-Lashof 1959]

Homogeneous fillers in s. sets: $\Lambda^{n,k} \to X$

means f sends the 2 "new" simplices to the same simplex modulo degeneracy



$[m] \underset{d}{\overset{s}{\rightleftarrows}} [n]$ $sd = 1$ $d^*s^* = 1$ ~~$\gamma \to s^*\gamma$~~

$\delta = s^*\gamma \Rightarrow d^*\delta = \gamma$, ∴ $\gamma \in \text{im } g \iff s^*\gamma \in \text{im } g$

pullback along g preserves i or converts i to an isom.

adjoin fillers, repeat, get $\tilde{X} \xrightarrow{\tilde{p}} \Gamma$;

its formation is compatible w. base change

Conj: if p is a qu-fib'n, then $\tilde{p}$ is a fib'n.

point out a lemma of Quillen from HAKT-I

lemma I a cat.

$$X : I \longrightarrow \text{Spaces}$$

$$\left| p \mapsto \coprod_{i_0 \to \cdots \to i_p \text{ in } I} X_{i_0} \right| \xrightarrow{p} BI$$

If X sends all arrows to h. eq's then p is a quasifibration

Example $I = [1] = \{0 < 1\}$ $\quad X_0 \xrightarrow{r} X_1$

get

$$\text{Map Cyl}(r) \xrightarrow{p} \Delta^1 \quad \text{is a qu-fib.}$$

now apply the conjecture to get

$$\widetilde{\text{Map Cyl}(r)} \xrightarrow{\tilde{p}} \Delta^1 \quad \text{a fib'n.}$$

but ~~omit~~ assume X_0, X_1 are fibrant, so we can omit fillers lying just over 0 or 1.

Observe the fibers are X_0, X_1, thereby proving Bousfield's fact w.o. the axiom of choice.