6.2

2. $V=\int_0^1 A(x)dx=\int_0^1 \pi (e^x)^2 dx = \frac{1}{2}\pi \int_0^1 e^{2x} d(2x) = \frac{1}{2}\pi e^{2x}\Big|_0^1 = \frac{\pi}{2}(e^2-1)$

4. $V=\int_2^5 A(x)dx=\int_2^5 \pi(x-1)dx = \pi\left(\frac{1}{2}x^2-x\right)\Big|_2^5 = \frac{15}{2}\pi$

6. $V=\int_0^1 A(y)dy=\int_0^1 \pi(y-y^2)^2 dy = \pi\int_0^1 (y^4-2y^3+y^2)dy$

$= \pi\left(\frac{1}{5}y^5-\frac{1}{2}y^4+\frac{1}{3}y^3\right)\Big|_0^1$

$= \frac{\pi}{30}$

8. $V=\int_{-1}^1 A(x)dx = \int_{-1}^1 [\pi(\sec(x))^2 - \pi(1)^2]dx$

$= 2\pi\int_0^1 (\sec^2 x - 1)dx$

$= 2\pi(\tan x - x)\Big|_0^1 = 2\pi(\tan 1 - 1)$

44 $V=\int_0^{10} A(x)\,dx \approx M_5 = \Delta x\,[A(1)+A(3)+\cdots+A(9)]$

$=2(0.65+0.61+0.59+0.55+0.50)$

$=5.80\ (m^3)$

52 $A(x)=(2y)^2=4(r^2-x^2)$

$V=\int_{-r}^{r} A(x)\,dx = 2\int_0^r 4(r^2-x^2)dx = 8\left(r^2x-\frac{1}{3}x^3\right)\Big|_0^r = \frac{16}{3}r^3$

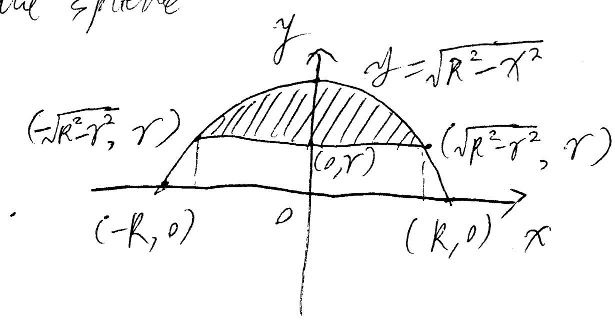
54. First the cross-section of the base corresponding to the coordinate y has length $2x = 2\sqrt{y}$. Thus we have area

$$A(y) = \frac{1}{2}\cdot\left(\frac{\sqrt{3}}{2}(2x)\right)\cdot(2x) = \frac{\sqrt{3}}{2}(2\sqrt{y})^2 = \sqrt{3}\cdot y$$

So,

$$V = \int_0^1 A(y)\,dy = \int_0^1 \sqrt{3}\cdot y\,dy = \sqrt{3}\left(\frac{1}{2}y^2\right)\Big|_0^1 = \frac{\sqrt{3}}{2}$$

66. The remaining portion of the sphere can be got by rotating the shaded region about the x-axis.



Thus, the volume is

$$V = \int_{-\sqrt{R^2-r^2}}^{\sqrt{R^2-r^2}} \pi\left[\left(\sqrt{R^2-x^2}\right)^2 - r^2\right]dx$$

$$= 2\int_0^{\sqrt{R^2-r^2}} \pi\,(R^2-x^2-r^2)\,dx$$

$$= 2\pi\left[(R^2-r^2)x - \frac{1}{3}x^3\right]_0^{\sqrt{R^2-r^2}}$$

$$= \frac{4\pi}{3}(R^2-r^2)^{\frac{3}{2}}$$

6.4

2. $F = mg = 60\cdot 9.8 = 588\ (N)$

$W = F\cdot d = 588\cdot 2 = 1176\ (J)$

6. We need find $f(x)$ first, $f(x) = k \cdot x$

Now we need k and we could get it from

$$25 = f(x) = kx = k(0.1),$$

so $k = 250$ (N/m). Thus $f(x) = 250x$.

5 cm = 0.05 m, so

$$W = \int_0^{0.05} 250x\,dx = 125x^2 \Big|_0^{0.05} = 0.3125 \text{ (J)}$$

10. We could set up two equations from the condition. Let's denote by L the natural length of the spring in meters.

$$6 = \int_{0.1-L}^{0.12-L} kx\,dx = \frac{1}{2}k[(0.12-L)^2 - (0.10-L)^2]$$

and

$$12 = \int_{0.12-L}^{0.14-L} kx\,dx = \frac{1}{2}k[(0.14-L)^2 - (0.12-L)^2]$$

Solve them, we get

$$k = 10000 \quad \text{and} \quad L = \frac{32}{400} \text{ (m)} = 8 \text{ (cm)}$$

12. We need consider the cable in two parts: One part is top 10 ft of the caple, which is lifted a distance x_i^* equal to its distance from the top. Second part, the remaining 30 ft of the cable is just lifted 10 ft.

$\frac{60}{40} = 1.5$ lb/ft is unit weight we need. Thus

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3}{2}x_i^* \Delta x + \frac{3}{2} \cdot 10\Delta x\right) = \int_0^{10} \left(\frac{3}{2}x + 15\right)dx = 525 \text{ (ft·lb)}$$

16. A horizontal cylindrical slice of water Δx ft thick has a volume of $\pi r^2 \cdot h = \pi \cdot 144 \cdot \Delta x$ (ft^3) and weights

$$(62.5\ lb/ft^3)(144\pi\ \Delta x\ ft^3) = 9000\pi\,\Delta x\ (lb).$$

If the slice lies x_i^* ft below the edge of the pool, then the work needed to pump it out is $9000\pi\, x_i^*\,\Delta x$. Thus

$$W = \lim_{n\to\infty}\sum_{i=1}^{n} 9000\pi\, x_i^*\,\Delta x = \int_1^5 9000\pi\, x\,dx$$

$$= 4500\pi x^2\Big|_1^5 = 108000\pi\ (ft\cdot lb)$$

6.5

2. $f_{ave} = \frac{1}{4-1}\int_1^4 \frac{1}{x}dx = \frac{1}{3}(\ln x)\Big|_1^4 = \frac{1}{3}\ln 4$

4. $g_{ave} = \frac{1}{4-1}\int_1^4 \sqrt{x}\,dx = \frac{1}{3}\left(\frac{2}{3}x^{\frac{3}{2}}\right)\Big|_1^4 = \frac{14}{9}$

6 $f_{ave} = \frac{1}{\frac{\pi}{4}-0}\int_0^{\frac{\pi}{4}} \sec\theta\tan\theta\,d\theta = \frac{4}{\pi}(\sec\theta)\Big|_0^{\frac{\pi}{4}} = \frac{4}{\pi}(\sqrt{2}-1)$