Math 120 - Fall 2001 - HW (Due 11.27.01) solutions:

§5.4

6. $\int \sqrt[3]{x}\,dx = \int x^{1/3}\,dx = \frac{x^{4/3}}{4/3} + C.$

8. $\int x(1+2x^4)\,dx = \int (x + 2x^5)\,dx = \frac{x^2}{2} + \frac{x^6}{3} + C.$

10. $\int \left(x^2 + 1 + \frac{1}{x^2+1}\right) dx = \frac{x^3}{3} + x + \tan^{-1} x + C$

46. $y = \sqrt[4]{x} \Rightarrow x = y^4$; Required int. $= \int_0^1 y^4\,dy = \frac{y^5}{5}\Big|_0^1 = \frac{1}{5}$

48. $\int_a^b I(t)\,dt = \int_a^b Q'(t)\,dt = Q(b) - Q(a)$ which is change in charge

50. $100 + \int_0^{15} n'(t)\,dt = 100 + n(15) - n(0) = 100 + n(15) - 100$

$= n(15) =$ population after 15 weeks.

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§5.5

6. $\int e^{\sin\theta}\cos\theta\,d\theta$.

$u = \sin\theta, \quad du = \cos\theta\,d\theta.$

$\hookrightarrow = \int e^u\,du = e^u + C = e^{\sin\theta} + C.$

8. $\int x^3(1-x^4)^5\,dx$: $1 - x^4 = u \Rightarrow du = -4x^3\,dx$

$\hookrightarrow = \int u^5 \cdot \frac{du}{-4} = \frac{u^6}{-24} + C = \frac{(1-x^4)^6}{-24} + C$

10. $\int (2-x)^6\,dx$: $2 - x = u \Rightarrow du = -dx.$

$\hookrightarrow = \int u^6(-du) = -u^7/7 + C = -\frac{(2-x)^7}{7} + C.$

12. $\int \frac{x}{x^2+1}\,dx$: $x^2 + 1 = u \Rightarrow \frac{du}{2} = x\,dx$

$\hookrightarrow = \int \frac{du}{2u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln(x^2+1) + C$

14. $\int x\,(x^2+1)^{3/2}\,dx \quad : x^2+1 = u \Rightarrow \frac{du}{2} = x\,dx$

$\hookrightarrow = \int \frac{u^{3/2}}{2}\,du.$

$= \frac{1}{2}\,\frac{u^{5/2}}{5/2} + c = \frac{1}{5}\,(x^2+1)^{5/2} + c.$

50. $\int_0^7 \sqrt{4+3x}\,dx$

$= \frac{(4+3x)^{3/2}}{\frac{3}{2}\cdot 3}\Big|_0^7 = \frac{2}{9}\left[(25)^{3/2} - (4)^{3/2}\right] = \frac{2}{9}\cdot 117 = 26$

52. $\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx \quad : x^2 = u \Rightarrow \frac{du}{2} = x\,dx$

$x = 0,\ u = 0\ ; \quad x = \sqrt{\pi},\ u = \pi$

$\hookrightarrow \int_0^{\pi} \frac{\cos u}{2}\,du = \frac{1}{2}\sin u\Big|_0^{\pi} = 0.$

54. $\int_0^{\pi/4} \sin 4t\,dt = \frac{-\cos 4t}{4}\Big|_0^{\pi/4} = \frac{-1}{4}(\cos\pi - \cos 0) = \frac{1}{2}.$

56. $\int_0^2 \frac{dx}{(2x-3)^2}$ → does not exist as $\frac{1}{(2x-3)^2}$ is discontinuous at $x = 3/2$

58. $\int_0^1 x e^{-x^2}\,dx \quad : -x^2 = u \Rightarrow \frac{du}{-2} = x\,dx$

$x = 0,\ u = 0; \quad x = 1,\ u = -1$

$\hookrightarrow \int_0^{-1} e^u \cdot \frac{du}{-2} = \frac{-1}{2}\,e^u\Big|_0^{-1} = \frac{-1}{2}\left(e^{-1} - e^0\right) = \frac{1}{2}(1 - 1/e)$