Solutions $\boxed{\text{HW } 10/11}$

Section 3.9

2 (a) $\tanh 0 = \dfrac{(e^0 - e^{-0})/2}{(e^0 + e^{-0})/2} = 0$

(b) $\tanh 1 = \dfrac{(e^1 - e^{-1})/2}{(e^1 + e^{-1})/2} = \dfrac{e^2-1}{e^2+1} \approx 0.76159$

4 (a) $\cosh 3 = \frac{1}{2}(e^3 + e^{-3}) \approx 10.06766$

(b) $\cosh(\ln 3) = \frac{1}{2}(e^{\ln 3} - e^{-\ln 3})$

$= \frac{1}{2}(3 + \frac{1}{3})$

$= \frac{5}{3}$

8. $\cosh(-x) = \frac{1}{2}[e^{-x} + e^{-(-x)}]$

$= \frac{1}{2}(e^x + e^{-x})$

$= \cosh x$

16 $\cosh^2 x + \sinh^2 x$

$= \left(\dfrac{e^x + e^{-x}}{2}\right)^2 + \left(\dfrac{e^x - e^{-x}}{2}\right)^2$

$= \frac{1}{4}(e^{2x} + 2 + e^{-2x}) + \frac{1}{4}(e^{2x} - 2 + e^{-2x})$

$$= \frac{1}{4}(2e^{2x} + 2e^{-2x})$$

$$= \frac{e^{2x} + e^{-2x}}{2}$$

$$= \cosh 2x$$

24. (a) $\frac{d}{dx}\cosh x = \frac{d}{dx}\left(\frac{1}{2}(e^{x}+e^{-x})\right)$

$$= \frac{1}{2}(e^{x} - e^{-x})$$

$$= \sinh x$$

(b) $\frac{d}{dx}\tanh x = \frac{d}{dx}\left(\frac{\sinh x}{\cosh x}\right)$

$$= \frac{\cosh x \cdot \cosh x - \sinh x \cdot \sinh x}{\cosh^2 x}$$

$$= \frac{1}{\cosh^2 x}$$

$$= \operatorname{sech}^2 x$$

(c) $\frac{d}{dx}\operatorname{csch} x = \frac{d}{dx}\left(\frac{1}{\sinh x}\right)$

$$= -\frac{1}{\sinh^2 x} \cdot \cosh x$$

$$= -\frac{1}{\sinh x} \cdot \frac{\cosh x}{\sinh x}$$

$$= -\operatorname{csch} x \cdot \coth x$$

(d) $\frac{d}{dx}\operatorname{sech} x = \frac{d}{dx}\left(\frac{1}{\cosh x}\right)$

$= -\frac{1}{\cosh x} \cdot \frac{\sinh x}{\cosh x}$

$= -\operatorname{sech} x \tanh x$

(e) $\frac{d}{dx}\coth x = \frac{d}{dx}\left(\frac{1}{\tanh x}\right)$

$= -\frac{1}{\tanh^2 x} \cdot (\tanh x)'$

$= -\frac{\cosh^2 x}{\sinh^2 x} \cdot \frac{1}{\cosh^2 x}$

$= -\frac{1}{\sinh^2 x}$

$= -\operatorname{csch}^2 x$

Comments : You also can get answer by $\frac{d}{dx}\left(\frac{\cosh x}{\sinh x}\right)$

26 Let $y = \cosh^{-1} x$. Then $\cosh y = x$,

so $\sinh y = \sqrt{\cosh^2 x - 1} = \sqrt{x^2 - 1}$.

so $e^y = \cosh y + \sinh y = x + \sqrt{x^2-1}$

Thus $y = \ln\left(x + \sqrt{x^2-1}\right)$

30. $f'(x) = 4 \cdot \operatorname{sech}^2 x$

32 $g'(x) = 2 \sinh x \cosh x$

34 $F'(x) = \cosh x \tanh x + \sinh x \operatorname{sech}^2 x$

36 $f'(t) = e^t \operatorname{sech} t - e^t \operatorname{sech} t \cdot \tanh t$

38 $f'(t) = \dfrac{1}{\sinh t} \cosh t = \coth t$

40 $y' = \cosh(\cosh x) \cdot (\cosh x)'$
$= \cosh(\cosh x) \cdot \sinh x$