Math 120-E - Solutions for hw due 10/02/2001

§ 3.3.

12. Soln: After 't' seconds, radius $r = 60t$.

$$A(t) = \pi(60t)^2 = 3600\pi t^2$$

$$A'(t) = 7200\pi t$$

i). $A'(1) = 7200\pi \text{ cm}^2/\text{s}$.

ii) $A'(3) = 21600\pi \text{ cm}^2/\text{s}$

iii) $A'(5) = 36000\pi \text{ cm}^2/\text{s}$.

32. a) If $dP/dt = 0$, population is stable or constant.

b). $\frac{dP}{dt} = 0$. Let $P(t) = P$

$$\Rightarrow \beta P(t) = r_0\left(1 - \frac{P(t)}{P_c}\right) \cdot P(t).$$

$$\Rightarrow \beta P/r_0 = 1 - P/P_c.$$

$$\Rightarrow \frac{P}{P_c} = \frac{-\beta}{r_0} + 1.$$

$$P = P_c(1 - \beta/r_0) = 10000(1 - 4/5) = \underline{2000}$$

c) If $\beta = 0.05$,

$P(t) = 0 \rightarrow$ no stable population.

§ 3.4

2. $f(x) = x \sin x$

Soln:

$$f'(x) = x\cos x + \sin x$$

4. $y = \cos x - 2\tan x$

Soln:

$$y' = -\sin x - 2\sec^2 x.$$

6. $g(t) = 4\sec t + \tan t$

Soln: $g'(t) = 4\sec t \tan t + \sec^2 t.$

8. $y = e^x \sin x$

Soln: $y' = e^x \cos x + e^x \cdot \sin x$

$$= e^x(\cos x + \sin x).$$

10. $y = \dfrac{\sin x}{1 + \cos x}$

Soln: $$y' = \frac{(1+\cos x)(\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}$$

$$= \frac{1 + \cos x}{(1+\cos x)^2} = \frac{1}{1+\cos x}.$$

12. $y = \dfrac{\tan x - 1}{\sec x}$

Soln: $y' = \dfrac{\sec x\,(\sec^2 x) - (\tan x - 1)(\sec x \tan x)}{\sec^2 x}$

$= \dfrac{\cancel{\sec x}(\overbrace{\sec^2 x - \tan^2 x}^{1} + \tan x)}{\cancel{\sec^2 x}\ \sec x}$,

$= \dfrac{1 + \tan x}{\sec x}$.

18. P.T. $\dfrac{d}{dx}(\sec x) = \sec x \tan x$.

Proof: $\sec x = 1/\cos x$

$\dfrac{d}{dx}(1/\cos x) = \dfrac{\cos x \cdot 0 - 1(-\sin x)}{\cos^2 x}$

$= \dfrac{\sin x}{\cos x} \cdot \dfrac{1}{\cos x} = \sec x \tan x.$

22. $y = 2\sin x$ at $(\pi/6, 1)$.

Soln: $y' = 2\cos x$; $y'\big|_{(\pi/6,1)} = 2 \cdot \cos \pi/6 = \sqrt{3}$.

Eqn: $y - 1 = \sqrt{3}(x - \pi/6) \Rightarrow y = \sqrt{3}x - \dfrac{\sqrt{3}\pi}{6} + 1$.

38. Find limit:

$$\lim_{\theta \to 0} \frac{\cos\theta - 1}{\sin\theta}$$

$$= \lim_{\theta \to 0} \frac{\frac{\cos\theta - 1}{\theta}}{\frac{\sin\theta}{\theta}}$$

$$= \frac{\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta}}{\lim_{\theta \to 0} \frac{\sin\theta}{\theta}} = \frac{0}{1} = \underline{\underline{0}}$$

40. $$\lim_{x \to 0} \frac{\tan x}{4x}$$

$$= \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{1}{4} \cdot \frac{1}{\cos x}$$

$$= \frac{1}{4} \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{1}{\lim_{x \to 0} \cos x}$$

$$= \frac{1}{4} \cdot 1 \cdot 1 = \underline{\underline{\frac{1}{4}}}$$