MATH 120 E, GRAYSON, EXAM #2, Oct 17, 2001

Name: ANSWERS Discussion Section: ________

No calculators. Simplification is optional.

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|---|---|---|---|---|---|---|---|
| 1 | 10 | 10 | 10 | 10 | 10 | 10 | |
| 2 | 6 | 12 | 10 | 6 | 6 | | |

Useful formulas:

$d/dx\ (\sin^{-1} x) = 1/\sqrt{1-x^2}$ $\qquad d/dx\ (\sinh^{-1} x) = 1/\sqrt{1+x^2}$

$d/dx\ (\cos^{-1} x) = -1/\sqrt{1-x^2}$ $\qquad d/dx\ (\cosh^{-1} x) = 1/\sqrt{x^2-1}$

$d/dx\ (\tan^{-1} x) = 1/(1+x^2)$ $\qquad d/dx\ (\tanh^{-1} x) = 1/(1-x^2)$

$\cosh^2 u - \sinh^2 u = 1$ $\qquad 1-\tanh^2 u = \operatorname{sech}^2 u$

1. (10 pts) Compute $\frac{d}{dx}((x^{33}+1)(x+1)^{33})$

$$(33x^{32})(x+1)^{33} + (x^{33}+1)\,33\,(x+1)^{32}$$

2. (10 pts) Compute $\frac{d}{dx}(\sin^{-1}\cosh x^2)$.

$$\frac{1}{\sqrt{1-(\cosh x^2)^2}} \cdot (\sinh x^2) \cdot 2x$$

3. (10 pts) Compute $\frac{d}{dx}\left(\frac{x}{\sqrt{x^2-1}}\right)$.

$$\left(\sqrt{x^2-1}\cdot 1 - x\cdot\tfrac{1}{2}(x^2-1)^{-1/2}\cdot 2x\right)\Big/(x^2-1)$$

4. (10 pts) Compute $\frac{d}{dx}(\log_2(1+x^2))$.

$$\frac{1}{\ln 2}\,\frac{1}{1+x^2}\cdot 2x$$

5. (10 pts) Compute $\frac{d}{dx}(2^x + x^2 + x^x)$.

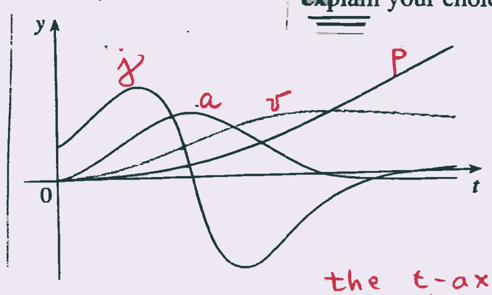
$$= \ln 2\cdot 2^x + 2x + \frac{d}{dx}\left(e^{x\ln x}\right)$$

$$= \ln 2\cdot 2^x + 2x + x^x\cdot\left(x\cdot\tfrac{1}{x} + 1\cdot \ln x\right)$$

6. (10 pts) Evaluate $\lim\limits_{x \to 1} \frac{\ln x}{x-1}$ by identifying it with a derivative $f'(a)$ of some function $f$ at some value $a$. State your choices. $f(x) = \ln x \quad a = 1$

The limit equals $\lim\limits_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a) = \frac{1}{a} = \frac{1}{1} = 1$

7. (6 pts) The figure shows the graphs of four functions. One is the position function of a car, one is the velocity of the car, one is its acceleration, and one is its jerk. Identify each curve, and explain your choices.




$j$ is the only function which is 0 when $a$ has its max, so $j = a'$. For the same reason, $a = v'$. $p$ is always increasing, so its derivative must be always positive, and $v > 0$, so $v = p'$. $j'$ would cross the t-axis twice, but no other curve does that so $j'$ is not there.

(This is 3.7 #4.)

8. (12 pts) Find the slope of the tangent line at the point (1, 2) to the curve defined by the equation $x^3 + y^3 - xy = 7$. Write down equations for the tangent line and normal line at that point.

$$3x^2 + 3y^2 y' - xy' - y = 0$$

$$(3y^2 - x) y' = y - 3x^2$$

$$y' = (y - 3x^2) / (3y^2 - x)$$

at $x = 1$ $y = 2$ get $(2 - 3)/(12 - 1) = -1/11$ for the slope

tang line : $y - 2 = -1/11 \, (x - 1)$

nml line : $y - 2 = 11 \, (x - 1)$

9. (10 pts) If a ball is thrown vertically upward with a velocity of 64 ft/s, then its height after $t$ seconds is $s = 64t - 16t^2$. What is the maximum height reached by the ball? What is the velocity of the ball when it is 48 ft above the ground on its way up? On its way down?

$s' = v = ds/dt = \text{velocity} = 64 - 32t$

$v = 0$ at the top, i.e. $64 = 32t$, so $t = 2$.

$s(2) = \text{max height} = 64 \cdot 2 - 16 \cdot 4 = 64$ ft

$s(1) = 48 = s(3)$

$s'(1) = 64 - 32 = 32$ ft/sec

$s'(3) = 64 - 32 \cdot 3 = -32$ ft/sec

(just like 3.3 # 8)

10. (6 pts) Show how to prove the product rule $\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx}$.

$$\frac{d}{dx}(uv) = \lim_{\Delta x \to 0} \frac{(u+\Delta u)(v+\Delta v) - uv}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta u\, v + u\, \Delta v + \Delta u\, \Delta v}{\Delta x}$$

$$= \left(\lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x}\right) v + u \lim_{\Delta x \to 0}\left(\frac{\Delta v}{\Delta x}\right) + \left(\lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x}\right)\left(\lim_{\Delta x \to 0} \frac{\Delta v}{\Delta x}\right)\left(\lim_{\Delta x \to 0} \Delta x\right)$$

$$= \frac{du}{dx} v + u \frac{dv}{dx} + \frac{du}{dx}\frac{dv}{dx} 0 = \frac{du}{dx} v + u\frac{dv}{dx}$$

11. (6 pts) Show how to prove that $\frac{d}{dx}(\cosh^{-1} x) = 1/\sqrt{x^2 - 1}$.

$y = \cosh^{-1} x \qquad x = \cosh y \qquad dx/dy = \sinh y$

$$dy/dx = 1/dx/dy = 1/\sinh y = 1/\sqrt{\cosh^2 y - 1}$$

$$= 1/\sqrt{x^2 - 1}$$