MATH 402, HOMEWORK 3

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More problems on Galois theory

Problem 14.5.6 (10 points). Let ζ_n denote a primitive n-th root of unity and let $K = \mathbb{Q}(\zeta_n)$ be the associated cyclotomic field. Let a denote the trace of ζ_n from K to $\mathbb{Q}$ (see exercise 18 of section 2 for the definition). Prove that $a = 1$ if $n = 1$, $a = 0$ if n is divisible by the square of a prime, and $a = (-1)^r$ if n is the product of r distinct primes.

Problem 14.5.10 (10 points). Prove that $\mathbb{Q}(\sqrt[3]{2})$ is not a subfield of any cyclotomic field over $\mathbb{Q}$.

Problem 14.5.12 (10 points). Let σ_p denote the Frobenius automorphism $x \mapsto x^p$ of the finite field $\mathbb{F}_q$, where $q = p^n$. Viewing $\mathbb{F}_q$ as a vector space of dimension n over $\mathbb{F}_p$ we can consider σ_p as a linear transformation. Determine its characteristic polynomial and prove it's diagonalizable iff $n \mid p - 1$.

Problem 14.6.29 (10 points).

Problem 14.6.31 (10 points).

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