MATH 402, HOMEWORK 1

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Problems from Section 4.5 on the Sylow theorems.

Problem 7 (10 points). Exhibit all Sylow 2-subgroups of S_4 and find elements of S_4 which conjugate one of these into each of the others.

Problem 13 (10 points). Prove that a group of order 56 has a normal Sylow p-subgroup for some prime p dividing its order.

Problem 16 (10 points). Let $|G| = pqr$, where p, q, and r are primes with $p < q < r$. Prove that G has a normal Sylow subgroup for either p, q, or r.

Problem 17 (10 points). Prove that if $|G| = 105$, then G has a normal Sylow 5-subgroup and a normal Sylow 7-subgroup.

Problems from Section 5.5 on semidirect products.

Problem 8 (10 points). Construct a non-abelian group of order 75. Classify all groups of order 75 (there are three of them).

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