Problem 1.

Suppose N is a normal subgroup of a group G, and $1 \to N \to G \to Q \to 1$ is an exact sequence of groups. Let p denote the map $G \to Q$.

Suppose there is a homomorphism $q : Q \to G$ such that $p \circ q = 1_Q$. Define a homomorphism $\phi : Q \to \mathrm{Aut}(N)$ by $\phi(\sigma) = [n \mapsto q(\sigma) \cdot n \cdot q(\sigma)^{-1}]$.

Show that G and q can be (re)constructed (up to isomorphism) from N, Q, and ϕ.